PhD thesis topic

Towards self-organisation in real systems: models and analysis

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The aim of this topic is to extend the current models of spatial self-organisation and discuss the significance of these extensions in real systems via a combination of analytical and numerical tools.

A typical theoretical approach is via Turing model of pattern formation [8, 7]. Nowadays, this model attracts a lot of attention from both theoretical perspective (where the implications of heterogeneity, growth, layering, stochasticity or delays just to name a few) and applications in real systems including chemical reactors or biology. Hence, it is now timely to investigate „structural stability“ of this formulation, meaning the stability of the model's behaviour to perturbations to the model's formulation itself.

A recent attempt in this sense was acknowledging the role of advection in real systems as *an effect of undergoing reactions in a given system*[4]. More recently, a rederivation of reaction-diffusion phenomenon in a mixture of substances suggests that an additional term, a square of concentration gradient, should be present in a reaction-diffusion model[3]. Note also the recent experimental observation that diffusion in a system undergoing reactions is not unrelated to the undergoing reactions [11].

As one branch (or perhaps as an initial step) of this thesis it is suggested to further analyse the effect of heterogeneity of reaction kinetics on the qualitative behaviour of a reaction-diffusion system. There are two avenues possible – first, rigorously showing that under certain (and which) conditions the formal WKBJ approach [6] yields correct estimation of the system's behaviour [5]; secondly, repeat the analysis in higher dimension. For the latter an inspiration from [2] is recommended.

As a parallel line of research, the objective would be to get acquainted with different frameworks of non-equilibrium thermodynamics [1, 9], rederive thermodynamically consistent reaction-diffusion models and analyse the difference to the classical models. Initial steps in this direction within linear non-equilibrium thermodynamics have been done in [10, 4, 3] where additional mechanism of transport was indicated (advection or a square of the concentration gradient; in the latter case, transient shock waves as in Burger's equations are suggested to appear). Further, preliminary results indicate that other extensions including a gradient of vorticity

(within the so-called GENERIC SHTC framework) might be present. All these extensions are yet unexplored in systems and only some understanding is available in initial and preliminary studies. Thus novel observations and results are expected.

Reference

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