Quantum Wheeler-DeWitt equations: interpretation, methods and toy models

(PhD project)

Every physicist knows that a consistent description of quantum gravity is currently by far the most important open problem and challenge for the theory. In this context, the build-up of the theory via the so called Wheeler-DeWitt (WDW) equation is one of the fairly promising strategies. The reasons are explained in chapter 9 of review [1] where the author emphasizes that the WDW equation might find the most natural mathematical as well as physical interpretation in the framework of quantum theory in the so called quasi-Hermitian (QH) formulation.

The latter expectations are updated in the more recent review [2]: We read there that "In the language of quantum phenomenology, the [model] may be perceived as representing a quantum catastrophe or collapse, resembling the quantum versions of phenomena such as Big Bang or Big Crunch in quantum gravity". After the recent developments in the field (see review [3]) the optimism got further strengthened, having rendered the study of the WDW equations a project accessible to ambitious doctorands.

Among the specific problems connected with such a project let us mention

(1) the question whether and under which conditions a unitary system can be specified via a set of non-Hermitian operators (of which the WDW Hamiltonian is a prominent special case).

(2) the necessity of development of suitable innovated mathematical methods (including, in particular, the construction of exceptional points or of the boundary of the domain of admissible parameters).

(3) the necessity of construction of tractable toy models (with one of the tools being an innovated perturbation theory).

References

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[1] A. Mostafazadeh, Int. J. Geom. Meth. Mod. Phys. 7 (2010) pp. 1191 - 1306.

[2] M. Znojil, "Non-self-adjoint operators in quantum physics: ideas, people, and trends", in Non-Selfadjoint Operators in Quantum Physics: Mathematical Aspects, edited by F. Bagarello et al, John Wiley & Sons, Hoboken, 2015, pp. 7 - 58.

[3] M. Znojil, Annals of Physics 385 (2017) pp. 162 - 179.

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