PhD thesis topic

Semigroups and non-equilibrium thermodynamics

doc. ing. Václav Klika, PhD.

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The aim of non-equilibrium thermodynamics is to present a rather general framework for formulation of thermodynamically consistent models. In an ideal world, we would like to guarantee the knowledge of mathematical well-posedness of the proposed models, which is almost perpendicular to the former approach (function spaces, boundary and initial conditions etc).

However, certain initial steps in the direction of combining these two approaches have been made. Namely:

- using Godunov-Boillat theorem (alternatively using Friedrichs, Lax approach) one can show that a certain type of physically relevant system of equations (a system of first order partial differential equations admitting an additional conservation law for a convex potential) is symmetric hyperbolic [6, 7, 3, 4, 1, 15, 9]; in addition, a converse statement is true
- from the classical analysis of partial differential equations the local-in-time well-posedness of the system, e.g. [14]
- a certain type of a system of symmetric hyperbolic partial differential equations has been shown to be thermodynamically consistent (SHTC) [5, 12] and within the GENERIC framework of non-equilibrium thermodynamics [11]

Hence the study of SHTC equations as a special case of equations compatible with non-equilibrium thermodynamics seems to be a good starting point for this thesis and the „mathematical formulation of nonequilibrium thermodynamics“.

Further, C_0 semigroups are a well-suited tool for studying well-posedness of evolution equations [10, 2]. In particular, the book by Rauch [14] is very close to the concept of semigroups and dissipative systems there and semigroups an hyperbolic systems are known to be related [13]. The suggestion is to try to use the theory of semigroups for showing well-posedness of SHTC equations as if this proves to be a successful approach, it would constitute a promising stepping stone for generalisation of these results to non-equilibrium thermodynamic approach.

As a particular initial creative goal, one could start with extending the results of [8, Chap2.9]. As an alternative path to follow it would be interesting to show the closeness of SHTC equations to Navier-Stokes equations as has been numerically shown[12]. Note that in SHTC equations there is only an algebraic dissipation representing viscosity as opposed to the classical approach where a Laplacian of velocity represents viscosity. The suggestion would be to use asymptotic methods for a small viscous term.

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