SHAPE OPTIMIZATION FOR QUANTUM SYSTEMS WITH MAGNETIC FIELDS

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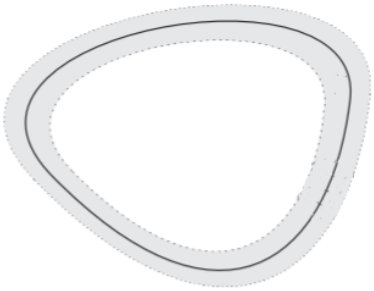
Type of work: bachelor/diploma project.

Description of the topic

The aim of the proposed project is to obtain optimization results for the ground-state energy in a family of model quantum systems with magnetic fields and non-trivial geometry. From the mathematical point of view, it means that there is a differential operator on a domain that describes the magnetic system and the goal is to show that its smallest eigenvalue attains the maximal or the minimal value for some specific shape. Typically, the optimal shape for the ground-state energy is radially symmetric [1, 3], and it can, for example, be the disk or the annulus. Optimization with magnetic fields is much less understood than without them and even very fundamental questions remain open [2], but it is certainly possible to select the setting, which is feasible to analyse within the bachelor or diploma project. The outcome of the project is expected to be a rigorous mathematical proof that certain shape optimizes the ground-state energy. On the level of the bachelor project partial results in this direction will also be sufficient.



One possible problem is to optimize the ground-state energy for the magnetic quantum system in such a curved strip built over a contour. The maximizer is expected to be the annulus.

References

[1] L. Erdős, Rayleigh-type isoperimetric inequality with a homogeneous magnetic field, *Calc. Var. PDE* **4** (1996), 283–292.

[2] S. Fournais and B. Helffer, Inequalities for the lowest magnetic Neumann eigenvalue, *Lett. Math. Phys.* **109** (2019), 1683–1700.

[3] A. Kachmar and V. Lotoreichik, On the isoperimetric inequality for the magnetic Robin Laplacian with negative boundary parameter, `arXiv:2108.05256`