PhD thesis topic

Analytic and geometric aspects of the Dirac equation

prof. Mgr. David KREJČIŘÍK, Ph.D., DSc. *

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Schrödinger operators have been extensively studied for more than a century and their spectral properties are well understood. The success is not only due to the existence of physical motivations associated with non-relativistic quantum mechanics over a long time period, but also due to the availability of powerful mathematical tools such as variational methods and the deep understanding of elliptic differential equations.

The spinorial structure and the lack of semiboundedness make their relativistic counterparts based on the Dirac equation much less understood. Notwithstanding, the discovery of graphene brought new motivations and focused attention to aspects of spectral analysis which had attracted little attention earlier. Among these there are complex electromagnetic fields [6], boundary perturbations [2, 3] and optimal shapes [1].

The objective of this PhD project is to contribute to this challenging area of mathematical physics by studying various analytic and geometric aspects in spectral theory of relativistic operators. In particular, the student will analyse criticality/subcriticality properties of the Dirac operator [5] and establish relativistic isoperimetric-type inequalities [4].

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References

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*Department of Mathematics, Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague; e-mail: david.krejcirik@fjfi.cvut.cz.